

Progression in Calculations at Shirley Warren Primary School

Written methods of calculations are based on mental strategies when working within the four rules of number. Written calculations involve jottings and informal methods initially before skills enable a child to progress on to more formal written methods of calculations.

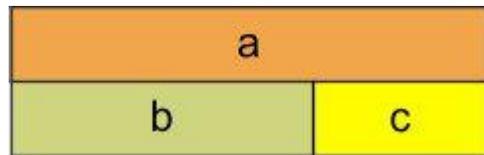
Strategies for calculation need to be supported by familiar models and images to reinforce understanding. Children understand strategies at different rates and the transition between stages should not be hurried. Within this booklet, progression of calculations is outlined in different year groups. Previous stages need to be revisited in order to consolidate understanding when beginning a new strategy.

Each of the stages in calculation we use within school are outlined in this booklet and explained through symbols, pictures and diagrams. To support all calculations children need to have a quick recall of number facts to 10, 20 and 100 as well as times table facts. Quick recall of number enables a child to progress through different calculations efficiently and provides greater access to problem solving.

Bar Model

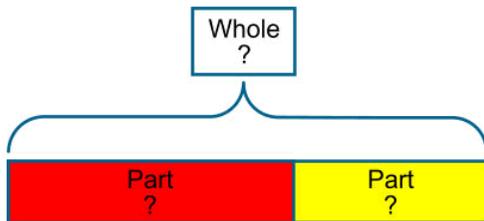
At Shirley Warren Primary School, we believe that children learn through concrete, pictorial and abstract strategies. We begin by learning using concrete objects such as the use of Numicon, multi-link and dienes. This allows all children to physically see what number is represented and allows them to move objects to make strong links with their calculation. From this stage children need to develop using pictures to aid their calculations but often this will not allow them to calculate the answer for the children, it will only reveal the mathematical structure of the problem. This pictorial stage bridges the gap between concrete and abstract representations.

The **bar model** supports understanding of the relationship between inverse operations and this is applied into other mathematical concepts.



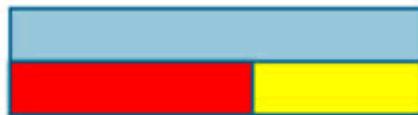
This image also helps us to identify different relationships that occur between numbers.

For example $a = b + c$; $a + c = b$; $a - b = c$; $a - c = b$



In problems, involving addition and subtraction there are three possible unknowns as seen to the left and given the value of two of them the third can be found.

The bar model can be rearranged to demonstrate equivalence in a traditional layout

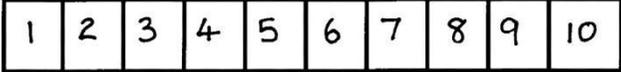
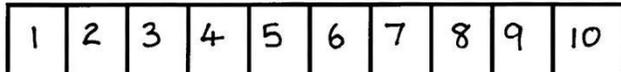
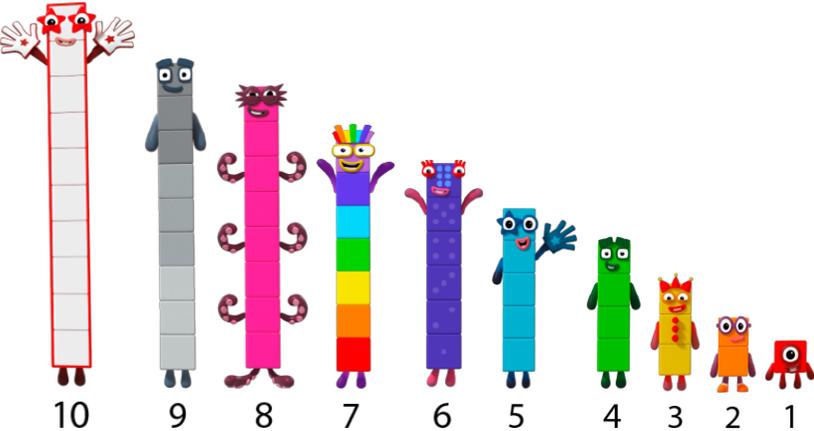


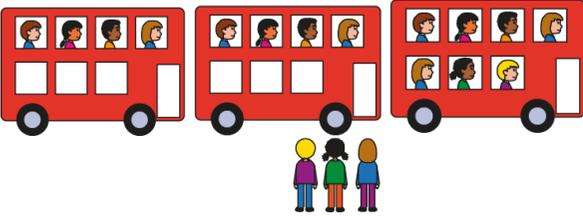
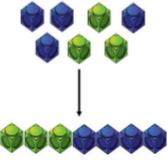
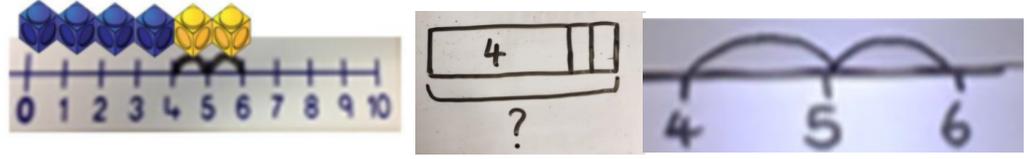
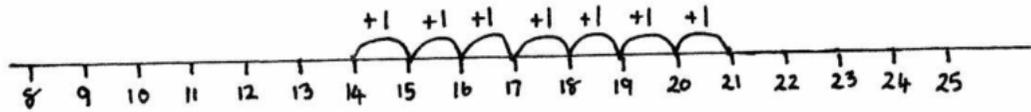
$$\begin{array}{|c|} \hline 6 \\ \hline \end{array} + \begin{array}{|c|} \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 10 \\ \hline \end{array}$$

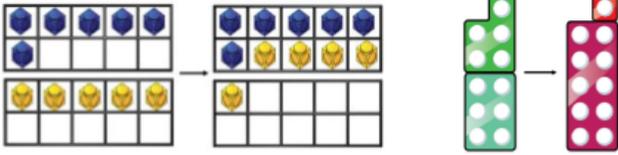
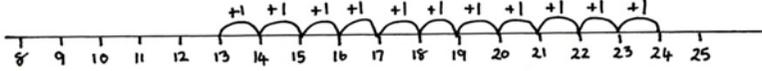
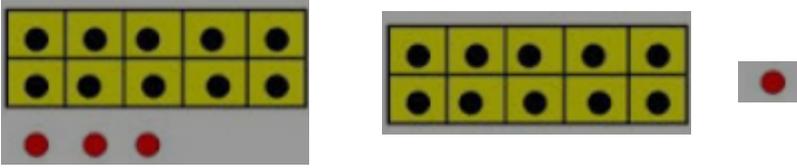
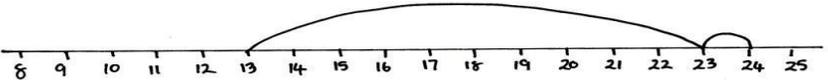
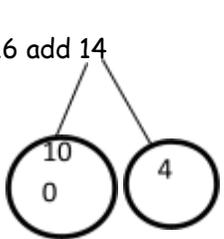
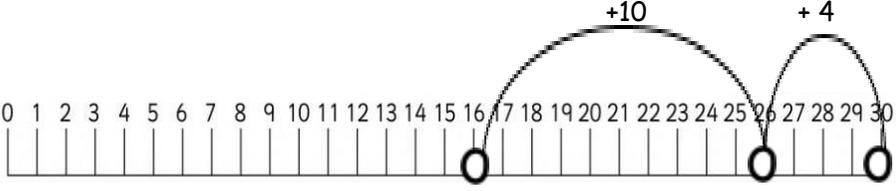
Pupils need to develop fluency in using this structure to represent the question (for example addition and subtraction problems) in a variety of contexts using the bar model. The model will help children to see that different problems share the same mathematical structure. This structure is versatile and its application can be used across children's development of number.

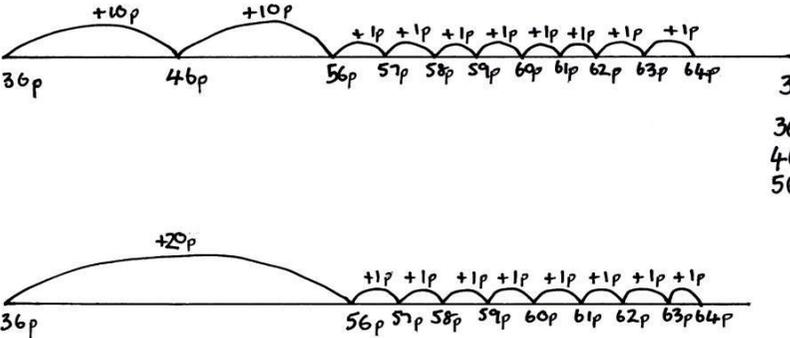
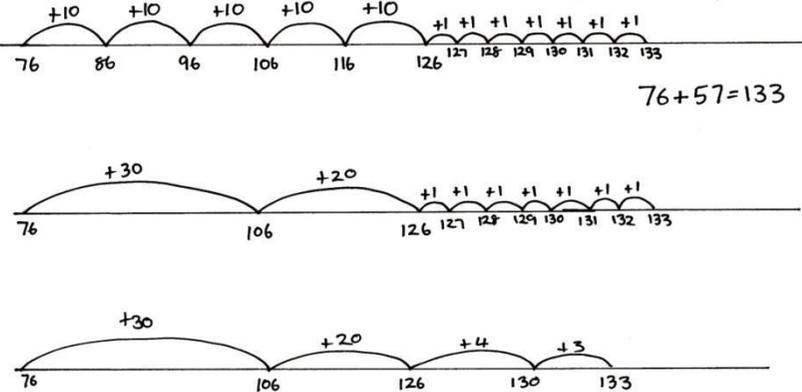
Descriptions and questions come courtesy of the NCETM.

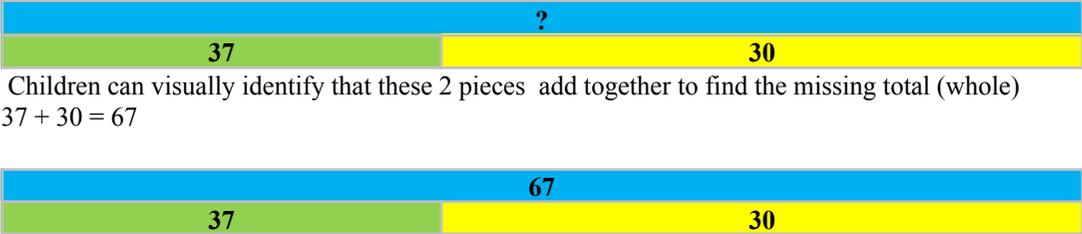
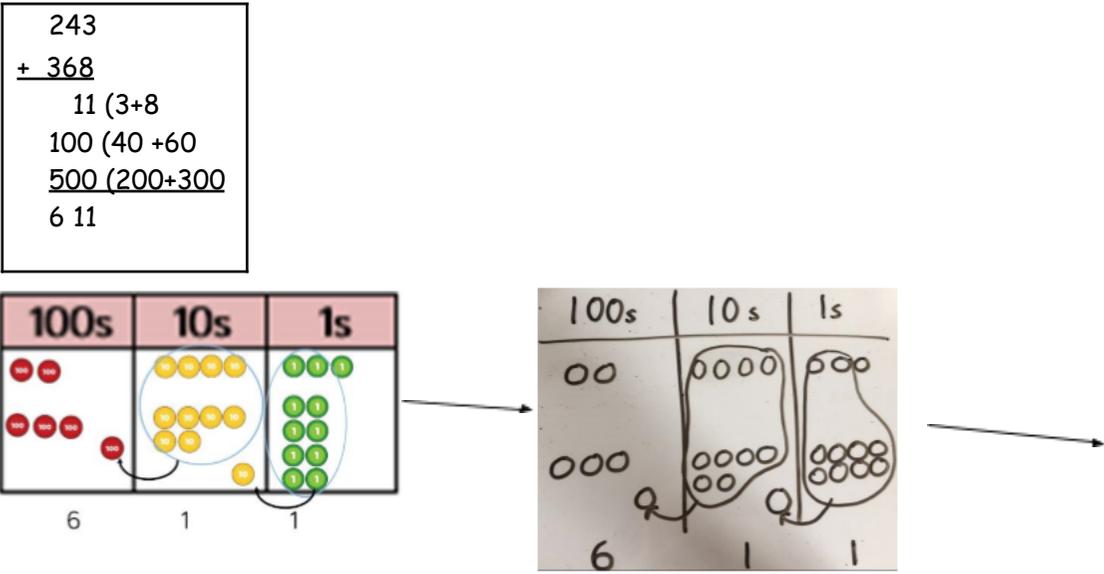
Progression in Addition

Year Group	Strategy	Examples	Vocabulary
	<p>Counting forwards in ones</p> <p>Hopping forwards on a number track</p>	  <p>Stand on 5 and hop forwards 1. What number are you on?</p> 	<p>Forwards, more than, numbers,</p>
	<p>Compose numbers</p>	 <p>Children will consider</p> <ul style="list-style-type: none"> • Which Numberblock is the shortest? Which is the tallest? 	

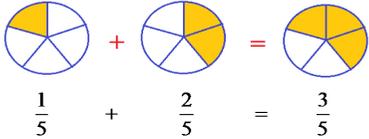
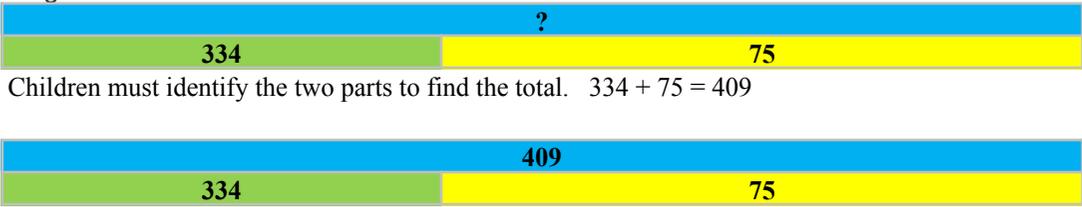
		<p style="text-align: center;">First Then Now</p>  <p style="text-align: center;">4 + 3 7</p> <p style="text-align: center;">4 + 3 → = 7</p>	
Combining two parts to make a whole		<p>3 green cube plus 4 blue cubes = 7 cubes</p> 	Add, more, altogether, total
<p>Year 1</p> <p>Using Numicon or objects to add on a number line</p> <p>Relate 'counting on' onto a number line</p>		<p>What is 2 more than 4?</p>  <p>If I have 14 apples and I find 7 more how many do I have?</p>  <p style="text-align: center;">$14 + 7 = 21$</p> <p style="text-align: center;">14 apples and 7 apples, 21 apples altogether.</p>	Add, more, altogether, total equals, add sign

<p>Year 2</p>	<p>Regrouping to add over ten</p> <p>Use structured number line to add a two digit number initially jumping in 1s, then 10s and 1s.</p>	<p>5+6 =</p>  <p>13+11</p>  <p>13 apples and 11 apples equals 24 apples.</p> <p>$13 + 11 = 24$</p>	<p>Partitioning, next, tens, number</p>
	<p>Use structured number line to add a two digit number jumping 10s and 1s</p>	<p>13+ 11 =</p>  <p>$13 + 10 = 23$ $+ 1 = 24$</p>  <p>13 apples and 10 apples, and 1 apple, equals 24 apples.</p> <p>$13 + 11 = 13 + 10 + 1$ $13 + 10 = 23$ $23 + 1 = 24$</p>	
<p>Year 2</p>	<p>Using the cherry method can ensure adding is efficient jumps. Within cherry method numbers are partitioned and added</p>	<p>14 + 16 =</p> <p>16 add 14</p>  	

Year 2	Use unstructured number line to add numbers when partitioning. (own numbering and jumps)	<p>I have 36p and my mum gives me 28p pocket money. How much money do I have altogether?</p>  <p> $36p + 28p = 64p$ $36 + 10 = 46p$ $46 + 10 = 56p$ $56 + 8 = 64p$ </p> <p> $36p + 28p = 64p$ $36 + 20p = 56$ $56 + 8p = 64$ </p>	Partitioning, efficient strategies
Year 2	Using unstructured number lines when crossing the hundreds boundary	 <p>$76 + 57 = 133$</p>	Partitioning, efficient strategy, next tens number
Year 2	Horizontal partitioning both numbers	<p>There are 23 cars in a showroom. 36 more arrive. How many are there in total?</p> $ \begin{array}{r} 23 \\ + 36 \\ \hline 9 \quad (3+6) \\ + 50 \quad (20+30) \\ \hline 59 \quad 50 + 9 \end{array} $	Partitioning Column addition - partitioning to the right
	Bar model allows children to use	Dylan has 37 coloured pencils and he buys 30 more. How many does he have now?	Total

<p>Year 2</p>	<p>pictorial images to support the understanding of abstract sums.</p>	 <p>Children can visually identify that these 2 pieces add together to find the missing total (whole) $37 + 30 = 67$</p>	<p>Add, Sum, Plus, Increase, Total</p>
<p>Year 3/4</p>	<p>Recognise fractions that total 1 or a whole</p>	 <p>$\frac{3}{4} + \frac{1}{4} = \frac{4}{4}$ or 1 whole</p>	<p>Fraction, Denominator Numerator whole</p>
<p>Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.</p>	<p>Concrete</p>	<p>pictorial</p> <p>Abstract</p>  <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-bottom: 10px;"> $\begin{array}{r} 243 \\ + 368 \\ \hline 11 \quad (3+8) \\ 100 \quad (40+60) \\ \hline 500 \quad (200+300) \\ 611 \end{array}$ </div>	<p>HTO 10s/ 1s column exchange</p>
<p>Year 3/4</p>	<p>This expanded method leads to a compact method</p>		<p>Partitioning</p>

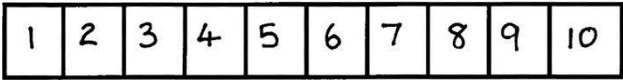
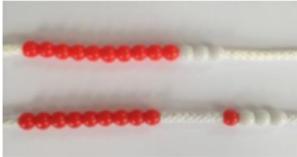
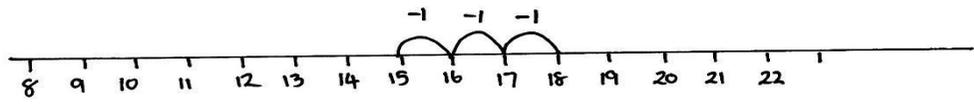
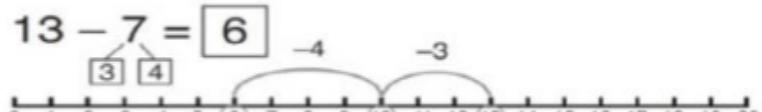
	<p>ensuring understanding of structure and efficiency. Recall of number facts is essential for efficiency.</p>	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin-bottom: 10px;"> $\begin{array}{r} 100\ 40\ 7 \\ + 200\ 40\ 8 \\ \hline 300\ 90\ 15 \\ 1 \end{array}$ </div> <p>There are 147 daffodil bulbs on a roundabout. 248 more daffodils are added. How many are there altogether now?</p> <div style="border: 1px solid black; padding: 5px; width: fit-content;"> $\begin{array}{l} 147 \\ + 248 \\ \hline 15\ (7+8) \\ 80\ (40+40) \\ \hline 300\ (200+100) \\ \hline 395\ (300 + 80 + 10 + 5) \end{array}$ </div> <div style="text-align: center; margin-top: 20px;">  </div>	<p>Column addition - partitioning to the right Column or vertical addition, adding ones first, adding units first, brackets, jottings carrying across</p>
<p>Year 3/ 4</p>	<p>Column method for addition</p>	<p>There are 123 cars in a showroom. 236 more arrive. How many are there in total?</p> $\begin{array}{r} 123 \\ + 236 \\ \hline 359 \end{array}$ <p>There are 359 cars in total</p> <p>There are 167 daffodil bulbs on a roundabout. 258 more daffodils are added. How many are there altogether now?</p> $\begin{array}{r} 167 \\ + 258 \\ \hline 425 \\ 11 \end{array}$	<p>Column method, vertical method, compact method, carry digits, carry forward, efficient method</p>
<p>Year 3/ 4</p>			

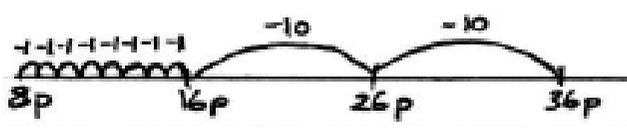
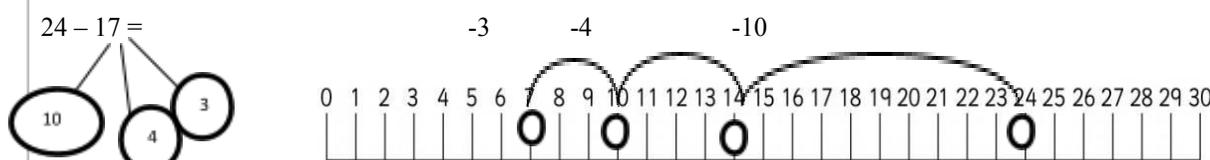
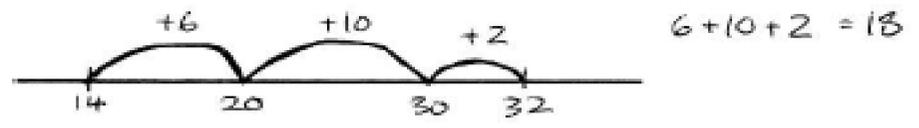
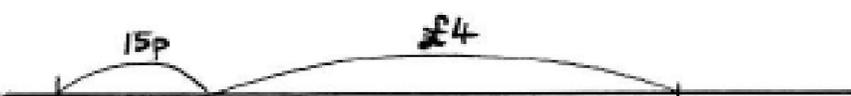
	<p>Build upon previous knowledge of expanded addition method when adding 4 digits by 4 digits.</p>	<p>There a 1234 flower in Mr Greens garden and 2340 in Mr Blooms. What is the total number of flowers in each both gardens?</p> $\begin{array}{r} 1234 \\ +2340 \\ \hline 4 \text{ (4+ 0)} \\ 70 \text{ (30 +40)} \\ 500 \text{ (300 +200)} \\ \underline{3000 \text{ (2000+1000)}} \\ 3574 \text{ (3000 + 500 + 70 + 4)} \end{array}$	<p>Total, partition ,column</p>
	<p>Add fractions with the same denominator</p>	 $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$	<p>Numerator, denominator, Fraction</p>
	<p>Bar model is still used as number increase depending on the year group</p>	<p>There are 334 children at Springfield School and 75 at Holy Trinity Nursery. How many children are there altogether?</p>  <p>Children must identify the two parts to find the total. $334 + 75 = 409$</p>	<p>Total, Whole Part, known, unknown</p>
<p>Year 5/6</p>	<p>Partitioned method for the addition of decimal number</p>	<p>Ann has to add two lengths of edging together. The strips measure 3.15m, 1.058m. What is the total length of edging</p> $\begin{array}{r} 3.15 \\ +1.058 \\ \hline 0.008 \text{ (0 +0.008)} \\ 0.1 \text{ (0 .05 + 0.05)} \\ 0.1 \text{ (0.1 +0)} \\ \underline{4 \text{ (4 + 0)}} \\ \underline{4.208 \text{ (4 + 0.1 + 0.1 + 0.008)}} \end{array}$	<p>Decimal point, Hundredths Thousandths</p>

	Column or vertical method for addition, with decimals	Ann has to add three lengths of edging together. The strips measure 3.15m, 1.058m and 0.8m. What is the total length of edging? $\begin{array}{r} 3.15 \\ +1.058 \\ \hline 4.208 \end{array}$ Then children must add the 0.8m Ann has 5.008m of edging.	Line up decimal point
Year 5/6	Column method for several numbers.	Bill owes Sarah £7702, Bob £77.02 and Stephanie £770.20. How much does he owe in total? $\begin{array}{r} 7702.00 \\ 77.02 \\ \hline 770.20 \\ \hline 8549.22 \end{array}$	Line up decimal point Hundredths Thousandths
Year 5/6	Bar model can visually allow students to visualise problems with multiple steps	Peter is playing Space Explorer on his computer. He finished 13 of the levels last week and 25 of the remaining levels this week. He has 12 more levels to complete. How many levels does Space Explorer have? 	
	Add fractions with different denominators	Marta filled a bucket with $\frac{2}{15}$ of a litre of water. A few minutes later, she added $\frac{3}{5}$ a litre of water How much water was within the bucket? $\frac{2}{15} + \frac{3}{5} = ?$ $\frac{2}{15} + \frac{3 \times 3}{5 \times 3}$ $\frac{2}{15} + \frac{9}{15} = \frac{2 + 9}{15} = \frac{11}{15}$ 	Numerator, denominator, Fraction

Progression in Subtraction

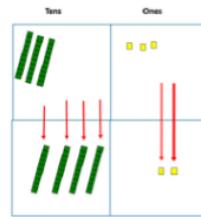
	Strategy	Examples	Vocabulary
	Practical activities through play	Role play activities – The shop has 5 apples and sells 1. How many apples are left in the shop?	Less, take away, one less, starting with, take away

			one, how many are left?
Year 1	Hopping back 1 and then more than 1 on a number track	<p>Stand on 5 and hop back 1. What number are you on?</p> 	Hop back, land on, 5 hop back 1 is 4
	Use concrete objects and fingers to support subtraction	<p>If a farmer has 13 apples on a tree and he picks 5. How many apples will be left?</p>  <p>Write a number sentences $13 - 5 = 8$</p>	Less, take away, less than, starting with, take away, how many are left?
	Represent a calculation with equipment	<p>If a farmer had 13 apples and he ate 4, how many would he have?</p> 	Jump back, hop back, land on, 18 take way 3 is 15
	Begin to record problems on a structured number line. Always start with the largest number to the right and subtract by jumping back to the left.	<p>If a farmer has 18 apples on a tree and he picks 3. How many apples will be left?</p>  <p><i>18 apples, take away 3 apples, equals 15 apples.</i></p> <p>$18 - 3 = 15$</p>	
Use knowledge of number bonds to solve subtraction problems by partitioning the second number	<p>$14 - 9 =$</p>  <p>Making 10</p> 	<p>Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.</p> <p>Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.</p>	Partition the second number

<p>Year 2</p>	<p>Use unstructured number line to subtract a two digit number from a two digit number jumping in 10s and 1s (Pre-requisite skill: counting on and back in steps of 10 and 1 from different starting numbers)</p>	<p>I have 36p, I spend 28p. How much do I have left?</p> <p>$36p - 28p = 8p$ You have 8p left.</p> 	<p>Start on 36 and hop/ jump back</p>
	<p>Children use the cherry method to partition numbers which will then allow them to subtract efficiently.</p>	<p>I have 24p and I spend 17p. How much do I have left?</p> <p>$24 - 17 =$</p> 	
	<p>Subtracting on an unstructured number line partitioning to the next tens number. Use multiples of 10 and 1s.</p>	<p>I have 36p, I spend 28p. How much do I have left?</p>  <p>When considering this mentally children should round 28 to 30 and then add 2 later on to ensure efficiency.</p>	<p>Partitioning, jump to the next tens number</p>
<p>Year 3/4</p>	<p>Partitioning without a number line. Partition the second number only.</p>	<p>There were 72 books in the library. Children borrowed 24 of them. How many books were left in the library?</p> <p>$72 - 24 = 72 - 20 - 4$ $72 - 20 = 52$ $52 - 4 = 48$. The answer is 48 books.</p>	<p>Partitioning</p>
	<p>Finding differences on an unstructured number line</p>	<p>In Class 6 there are 32 children, and 14 in Class 5. What is the difference in class size between Class 6 and Class 5?</p>  <p>The difference is 18 children.</p> <p>Beth goes shopping with £5. She spends 85p. How much change will she get?</p>  <p>She will get £4.15 change.</p>	<p>Difference, counting on, counting up</p>

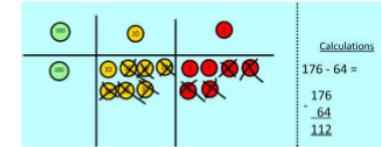
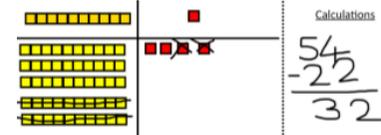
Year 3/4

Subtraction without regrouping



Use Base 10 to make the bigger number then take the smaller number away.

Draw the Base 10 or place value counters alongside the written calculation to help to show working



A child has 74 marbles and gives 23 away. How many are left?

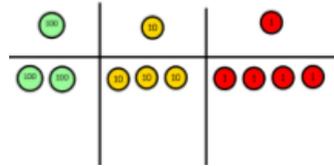
$$\begin{array}{r} 70 \ 4 \\ - 20 \ 3 \\ \hline 50 \ 1 \end{array}$$

So the answer is 51 marbles

Partitioned column with exchanging

Expanded subtraction method

*This is a big step and children need a lot of reinforcement on place and number value and checking answers



Calculations

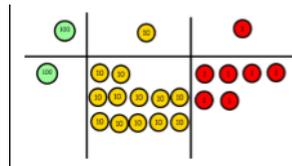
$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

Make the larger number with the place value counters

Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.

Now take away the ones

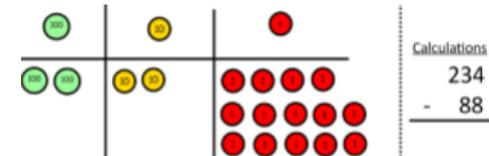


Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

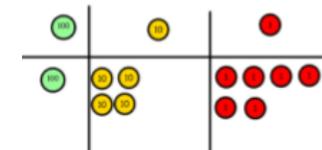
Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.

Now I can take away eight



Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$



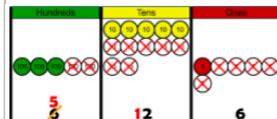
Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline 146 \end{array}$$

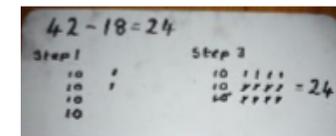
tens and complete my subtraction

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount

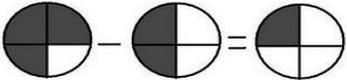
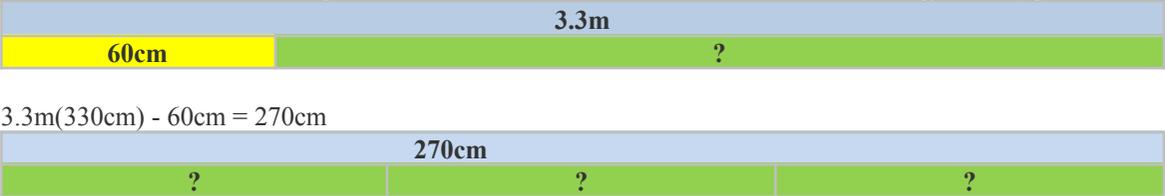
Pictorial



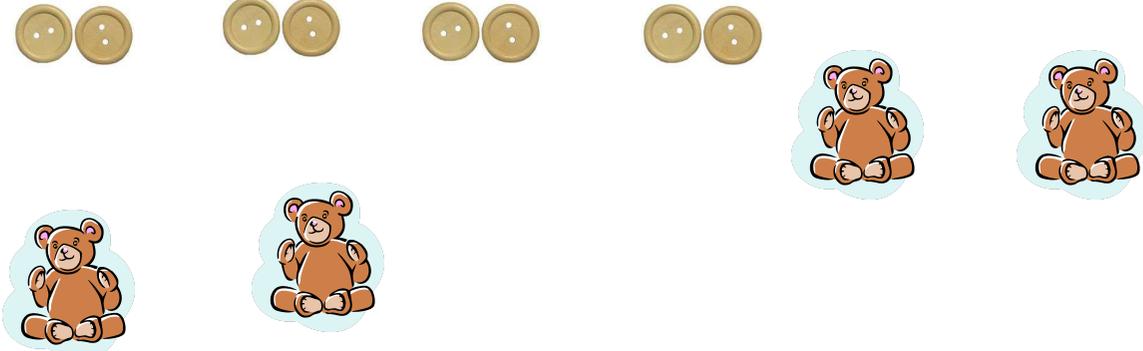
Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

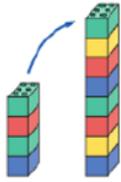
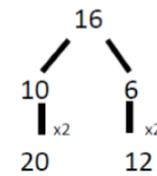
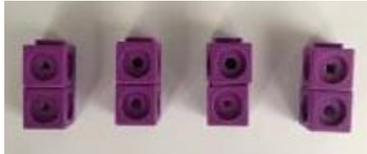
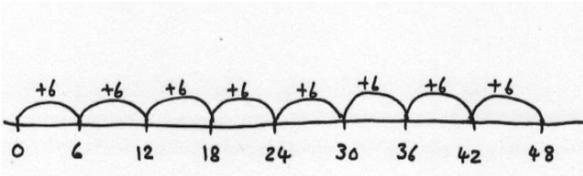


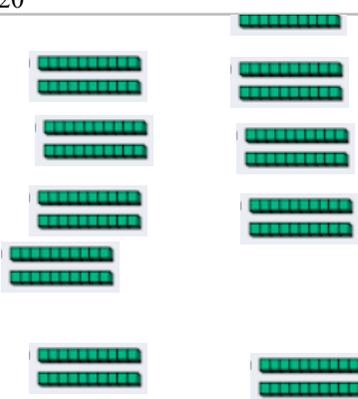
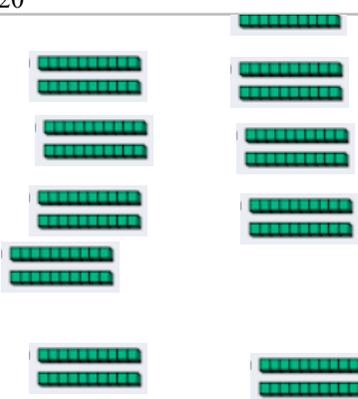
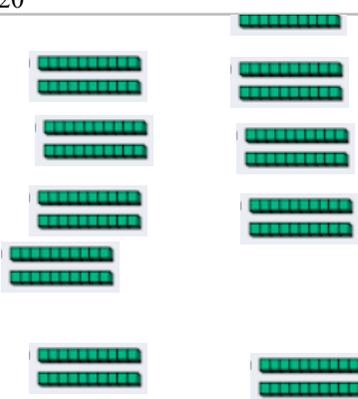
Partition, expanded method, exchange Borrow

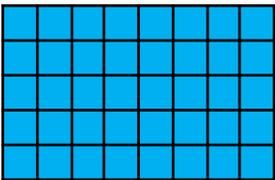
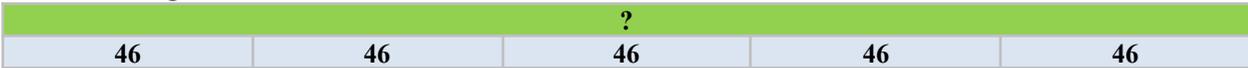
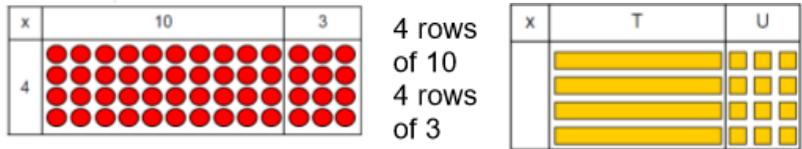
		$\begin{array}{r} - 186 \\ 156 \end{array}$	
Year 3/4	Children will be able to subtract fractions with the same denominator	 $\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$	
Year 5 /6	Column or vertical method with decimals	<p>A plant root measures 23.062 cm. As part of an experiment a scientist cuts off 1.0005cm. How much root is left?</p> $\begin{array}{r} 23.0620 \\ - 1.0005 \\ \hline 22.0615 \end{array}$ <p>22.0615cm of root is left (22.06cm rounded to 2 decimal places)</p>	Exchanging, concise column or vertical method
	Children will be able to subtract fractions with common denominators	$\frac{3}{4} - \frac{1}{3} = \frac{3 \times 3}{4 \times 3} - \frac{1 \times 4}{3 \times 4}$ $= \frac{9}{12} - \frac{4}{12}$ <p>If there is $\frac{3}{4}$ of a pizza left and I eat $\frac{1}{3}$ of it. How much pizza will remain?</p> $= \frac{5}{12}$	fraction, proper/improper fraction mixed number numerator, denominator
Year 5/6	Children can use the bar model to solve 2 step problems which uses different operations.	<p>I cut 60 cm from 3.3m of string and shared the rest between 3 friends. How much string did they get each?</p>  <p>3.3m(330cm) - 60cm = 270cm</p> <p>270cm divided by 3 = 90cm</p>	Exchange Borrow Split, Total,
Year 5/6	Solving multi step problems in context that require children to consider the order the problem should be solved	<p>I have read 213 pages of my 1004 page book. How many pages must I read until I reach the middle?</p> <p>Children should mentally half 1004 = 502</p> $\begin{array}{r} 4 \overline{) 1004} \\ \underline{213} \\ 289 \end{array}$	Half Subtract Less
	Subtraction fraction that have different denominators.	$\frac{3}{4} - \frac{1}{3} = \frac{3 \times 3}{4 \times 3} - \frac{1 \times 4}{3 \times 4}$ $= \frac{9}{12} - \frac{4}{12}$	fraction, proper/improper fraction

Progression in Multiplication

Year Group	Strategy	Examples	Vocabulary
Year 1	Count in multiples.	<p>Children should be able to count in multiples of 2, 5 and 10 There are 2 wheels on each bike how many altogether?</p> 	
Year 1	Grouping and 'lots of' with concrete materials and recording using pictures	<p>Each teddy has two buttons. Draw the buttons on the teddy bears. How many buttons is that altogether?</p> 	Group, lots of, altogether

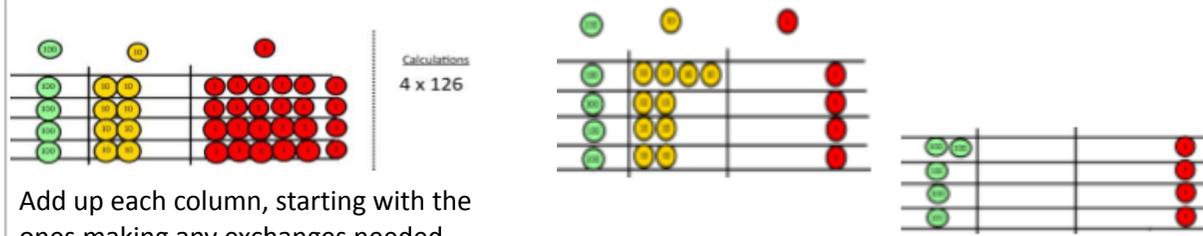
	<p>Pre-multiplication: Understand the idea of arrays: 2 rows with 3 eggs in each row 3 columns with 2 eggs in each column</p>	<p>2 buttons and 2 buttons and 2 buttons and 2 buttons is 8 buttons</p> <p>Noticing arrays in everyday objects e.g. chocolates in a box, windows in a building - use the language of arrays.</p>  <p>2 rows of 3 eggs.</p>	<p>Rows of, arrays, columns</p>
<p>Year 2</p>	<p>Double and halve numbers of multiples of 5.</p>	<p>Mentally- Double 25 = 50</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>  <p>Double 4 is 8</p>  <p>16 10 6 x2 x2 20 12</p> <p>Partition a number and then double each part before recombining it back together.</p> <p>Represent or draw the number and create it again before adding together</p>	<p>Double Lots of</p>
<p>Year 2</p>	<p>Understand multiplication as 'groups of', equal jumps</p> <p>6x8 } 6 multiplied by 8 } 8 lots of 6 } -all meaning the same</p>	  <p>Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30</p> <p>There are six carrots in a bag. How many carrots are there in eight bags?</p> <p>$6 \times 8 = 48$ There are 48 carrots altogether.</p> 	<p>Groups of, altogether, counting forwards, equal steps, equal jumps</p>
	<p>Arrays showing communitive values</p>	  <p>$4 \times 2 = 8$ $2 \times 4 = 8$</p>  <p>$5 + 5 + 5 = 15$</p>  <p>$3 + 3 + 3 + 3 + 3 = 15$</p>	<p>Groups of, altogether, counting forwards, equal</p>

			steps, equal jumps area																			
Year 3	Count in multiples of 4, 8 50 and 100 from 0.	Mentally- count in multiples of 4, 8, 50 and 100 using their knowledge of counting in 2s, 5s and 10s																				
	Double any number to 50	Mentally- Can you double 34? 2 lots of 30 = 60 2 lots of 4 =8 60+8=68	<div style="border: 1px solid black; padding: 10px; width: fit-content; margin: auto;"> Represent each number once (either with equipment or drawings) and duplicate this ten times </div>																			
Multiplying by 10	<table border="1" style="margin-bottom: 10px;"> <tr> <td style="width: 10%;"></td> <td style="width: 30%;">x</td> <td style="width: 30%;">20</td> <td style="width: 30%;">1</td> </tr> <tr> <td></td> <td>10</td> <td>  </td> <td>  </td> </tr> </table> <table border="1" style="margin-bottom: 10px;"> <tr> <td style="width: 10%;"></td> <td style="width: 15%;">H</td> <td style="width: 15%;">T</td> <td style="width: 15%;">O</td> </tr> <tr> <td></td> <td></td> <td>2</td> <td>1</td> </tr> <tr> <td></td> <td>2</td> <td>1</td> <td>0</td> </tr> </table> <p>Showing using a place value grid once this has been demonstrated both with concrete resources and drawings</p>		x	20	1		10				H	T	O			2	1		2	1	0	Horizontal partitioning, combine answers
	x	20	1																			
	10																					
	H	T	O																			
		2	1																			
	2	1	0																			

<p>Learning the Commutative law to allow children to work with calculation they are more comfortable with.</p>	 <p>There are 8 bottles kept within cases. If I have 5 full cases, how many bottles do I have?</p> $8 \times 5 = 40$ $5 \times 8 = 40$ $8 \times 5 = 5 \times 8$	<p>Multiply Times multiple of, product</p>
<p>Double numbers to calculate larger multiples</p>	$7 \times 2 = 14$ $7 \times 4 = 28$ $7 \times 8 = 56$ $7 \times 16 = 112$	<p>double, halve share, share equally</p>
<p>When using Bar model children will identify that each part is the same and then they are creating a total. This is a strong link to repeated addition to find a total</p>	<p>A bookcase in the library holds 5 shelves with 46 books on each shelf. How many books are there in the bookcase altogether?</p>  <p>$46 + 46 + 46 + 46 + 46 = 230$ or $5 \times 46 = 230$</p>	
<p>Grid method</p>	 <p>4 rows of 10 4 rows of 3</p> <p>4 rows of 13</p> <p>Show the link with arrays to first introduce the grid method.</p> <p>Move on to using Base 10 to move towards a more compact method</p>	
<p>Year 4 Count in multiples of 6, 7, 9, 25 and 1000</p>	<p>Mentally- count in all multiples to 10 using their prior knowledge of multiples.</p>	
<p>Double numbers beyond 100.</p>	<p>Mentally- double amounts beyond 100 using partitioned method.</p> <p>Double 162 $100 \times 2 = 200$</p> $60 \times 2 = 120 +$ $2 \times 2 = 4$ <hr style="width: 100px; margin-left: 100px;"/> <p>Total 324</p> <p>Start to double decimal numbers in the context of money. £1.20 doubled is £2.40</p>	<p>Double Multiply</p>

Year 4

Grid method multiplication



Add up each column, starting with the ones making any exchanges needed.

My family of 4 is going to a theme park. The cost of each ticket will £124. How much will it cost the whole family?

X	100	20	4	Total
4	400	80	16	496

Tickets cost £25 each for a concert. Eighteen people buy a ticket. How much money is taken on ticket sales?

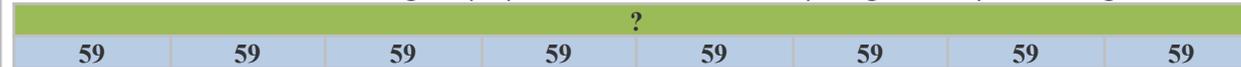
X	20	5	Total
10	200	50	250
8	160	40	200
			450

Thinking: $8 \times 20 = (8 \times 2) \times 10$

$25 \times 18 = £450$ was taken in ticket sales.

Through the use of the bar model, children can represent amount within the bottom of the bar chart in the correct number of sections. For example when multiplying by 5 children will have 5 boxes below the total.

8 children each download 59 songs to play on their iPod. How many songs do they have altogether?



Children draw 8 boxes and they know each section has 59 songs. Children can then use grid method to calculate $8 \times 59 =$

x	50	9	Total
8	400	72	472

$400 + 72 = 472$

Grid method, multiply, totals, area, associated/linked multiplication facts

Place value.
Divide
Multiply
Move the digits
Product

Sections
Total
Amounts,
Bars

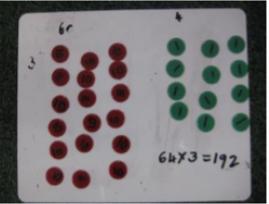
Year 5

Double numbers with 2 decimal places.

Mentally- double decimal values through partioned methods.

£3.45 doubled £3 x 2 = 6 £0.4/40p x 2 = £0.8 or 80p £0.05/5p x 2 = £0.1/10p

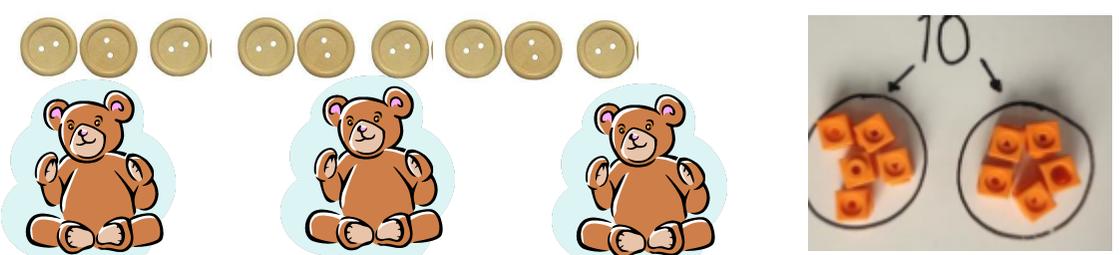
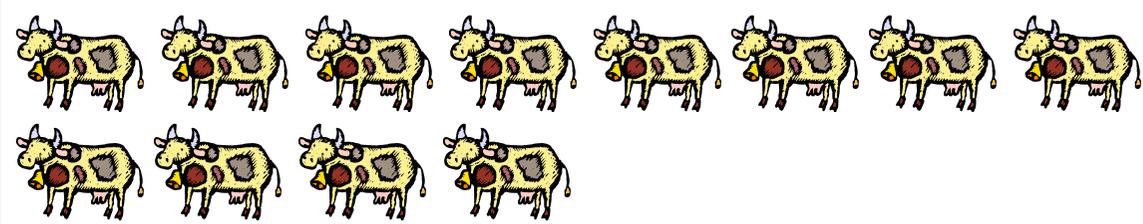
Total £6 + 80p +10p = £6.90

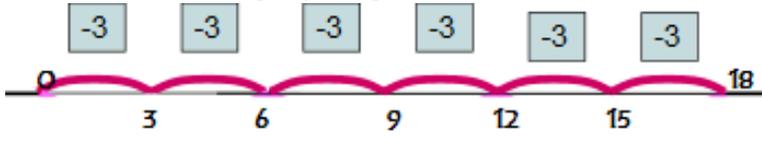
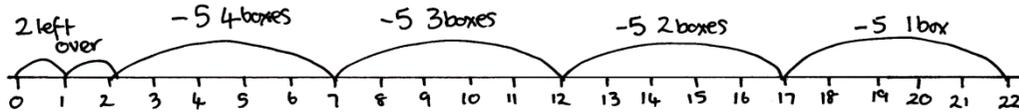
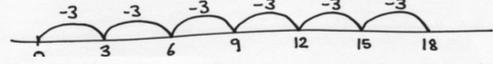
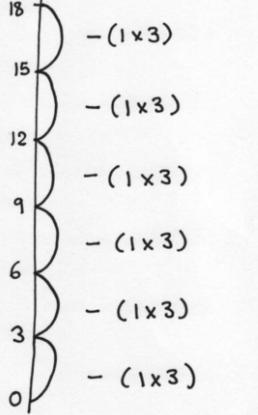
	<p>Simple expanded multiplication</p>	<p>Children can continue to be supported by place value counters at the stage of multiplication.</p>  <p>It is important at this stage that they always multiply the ones first and note down their answer followed by the tens that they note below.</p> <p>There are 275 bottles of lemonade drunk each day for 8 days. How many bottles of lemonade are drunk altogether?</p> $ \begin{array}{r} 275 \\ \times 8 \\ \hline 40 \quad (5 \times 8) \\ 560 \quad (70 \times 8) \\ \underline{1600} \quad (200 \times 8) \\ \underline{2200} \end{array} $	<p>Simple expanded method columns</p>																				
<p>Year 5</p>	<p>Multiplication with decimals</p> <p>Use partitioning of number and known tables facts</p>	<table border="1" data-bbox="705 512 981 635"> <tr> <td></td> <td>4</td> <td>0.9</td> <td>0.02</td> </tr> <tr> <td>3</td> <td>12.00</td> <td>2.70</td> <td>0.06</td> </tr> </table> $ \begin{array}{r} 12.00 \\ + 2.70 \\ \underline{0.06} \\ 14.76 \end{array} $		4	0.9	0.02	3	12.00	2.70	0.06													
	4	0.9	0.02																				
3	12.00	2.70	0.06																				
<p>Year 5</p>	<p>Expanded method- representing the grid method in a column format</p> <p>Use rounding to approximate the answer</p> <p>Be able to jot down the stages to the right</p> <p>Know that the 'one' in 18 is not a one but a ten!</p>	<p>Cakes weigh 125g each. Dave has 18 cakes, how much do they weigh altogether?</p> <table border="1" data-bbox="629 727 1077 970"> <tr> <td>x</td> <td>10</td> <td>8</td> <td></td> </tr> <tr> <td>100</td> <td>1000</td> <td>800</td> <td>1800</td> </tr> <tr> <td>20</td> <td>200</td> <td>160</td> <td>360</td> </tr> <tr> <td>5</td> <td>50</td> <td>40</td> <td>90</td> </tr> <tr> <td></td> <td></td> <td></td> <td><u>2250</u></td> </tr> </table> <p>Approximate = 125 x 18 is approximately equal to 125 x 20 125 x 20 = 2500 (125 x 10 = 1250 then double which is 2500)</p> $ \begin{array}{r} 125 \\ \times 18 \\ \hline 1000 \quad 125 \times 8 \\ \underline{1250} \quad 125 \times 10 \\ \underline{2250} \quad \text{Check against approximation OK!} \\ 125 \times 18 = 2250 \end{array} $ <p>The cakes weigh 2250g altogether.</p>	x	10	8		100	1000	800	1800	20	200	160	360	5	50	40	90				<u>2250</u>	<p>Expanded method, vertical, column, rounding, units of measure</p>
x	10	8																					
100	1000	800	1800																				
20	200	160	360																				
5	50	40	90																				
			<u>2250</u>																				

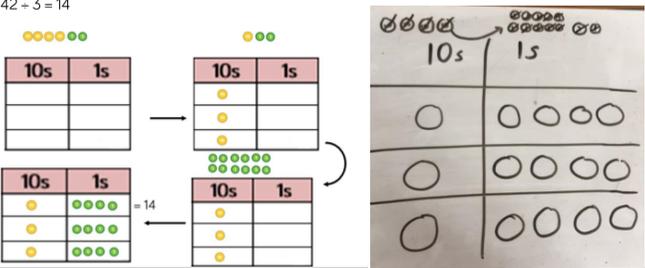
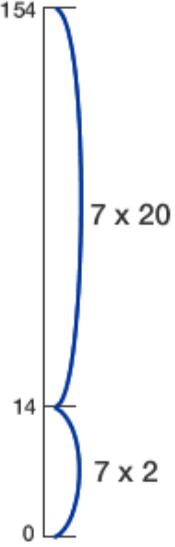
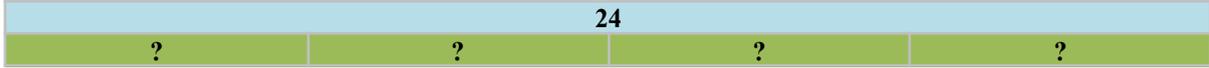
	Children continue to use the bar model to solve word problems of more complexity.	<p>Within a football stadium there are 7 stands. 467 children can sit in each stand. What is the total number of seats in the stadium?</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td colspan="7" style="background-color: #92d050; text-align: center;">?</td></tr> <tr><td style="background-color: #add8e6;">467</td><td style="background-color: #add8e6;">467</td><td style="background-color: #add8e6;">467</td><td style="background-color: #add8e6;">467</td><td style="background-color: #add8e6;">467</td><td style="background-color: #add8e6;">467</td><td style="background-color: #add8e6;">467</td></tr> </table> <p>Children split the lower bar into the second bar into 7 sections, with each part (stadium) seating 467.</p>	?							467	467	467	467	467	467	467	Sections Total Amounts, Bars																					
?																																						
467	467	467	467	467	467	467																																
Year 5	<p>Column method</p> <p>The 'carry' digits can either be written in or carried mentally</p>	<p>A school visit costs £326 per person. If sixty seven people sign up for the visit how much do they pay altogether?</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: right;">326</td> <td style="text-align: right;">326 x 67 is approximately equal to 300 x 7</td> <td style="text-align: right;">300 x 70 = 21000</td> </tr> <tr> <td style="text-align: right;">X 67</td> <td></td> <td></td> </tr> <tr> <td style="text-align: right;">42 7x6</td> <td style="text-align: right;">326</td> <td></td> </tr> <tr> <td style="text-align: right;">140 7 x 20</td> <td style="text-align: right;">X 67</td> <td></td> </tr> <tr> <td style="text-align: right;">2100 7 x 300</td> <td style="text-align: right;">2282</td> <td></td> </tr> <tr> <td style="text-align: right;">360 60 x 6</td> <td style="text-align: right;"><u>19560</u></td> <td></td> </tr> <tr> <td style="text-align: right;">1200 60 x 20</td> <td style="text-align: right;"><u>21842</u></td> <td style="text-align: right;">Check against approximation!</td> </tr> <tr> <td style="text-align: right;"><u>18000</u> 60 x 300</td> <td></td> <td></td> </tr> <tr> <td style="text-align: right;"><u>21842</u></td> <td style="text-align: right;">326 x 67 = 21 842</td> <td style="text-align: right;">They pay £21 842 altogether.</td> </tr> </table>	326	326 x 67 is approximately equal to 300 x 7	300 x 70 = 21000	X 67			42 7x6	326		140 7 x 20	X 67		2100 7 x 300	2282		360 60 x 6	<u>19560</u>		1200 60 x 20	<u>21842</u>	Check against approximation!	<u>18000</u> 60 x 300			<u>21842</u>	326 x 67 = 21 842	They pay £21 842 altogether.	Column or vertical method lots of, groups of times, multiplication, multiply, multiplied by multiple of, product								
326	326 x 67 is approximately equal to 300 x 7	300 x 70 = 21000																																				
X 67																																						
42 7x6	326																																					
140 7 x 20	X 67																																					
2100 7 x 300	2282																																					
360 60 x 6	<u>19560</u>																																					
1200 60 x 20	<u>21842</u>	Check against approximation!																																				
<u>18000</u> 60 x 300																																						
<u>21842</u>	326 x 67 = 21 842	They pay £21 842 altogether.																																				
Year 6	<p>Short multiplication of 2, 3 and 4 digit numbers by a 1 digit number,</p>	<p>A family of 8 want to go on holiday to Spain. It will cost £ 463 for each person. What is the total cost for all 8 family members.</p> <p>Children should remember to calculation of an approximation before calculating their answer.</p> <table style="margin-left: auto; margin-right: auto;"> <tr><td></td><td style="text-align: center;">H</td><td style="text-align: center;">T</td><td style="text-align: center;">U</td><td></td></tr> <tr><td></td><td style="text-align: center;">4</td><td style="text-align: center;">6</td><td style="text-align: center;">3</td><td></td></tr> <tr><td style="text-align: right;">x</td><td></td><td></td><td style="text-align: center;">8</td><td></td></tr> <tr><td colspan="5" style="border-top: 1px solid black;"></td></tr> <tr><td></td><td style="text-align: center;">3</td><td style="text-align: center;">7</td><td style="text-align: center;">0</td><td style="text-align: center;">4</td></tr> <tr><td colspan="5" style="border-top: 1px solid black;"></td></tr> <tr><td></td><td style="text-align: center;">5</td><td style="text-align: center;">2</td><td></td><td></td></tr> </table>		H	T	U			4	6	3		x			8								3	7	0	4							5	2			lots of, groups of times, multiplication, multiply, multiplied by multiple of, product
		H	T	U																																		
	4	6	3																																			
x			8																																			
	3	7	0	4																																		
	5	2																																				
Preform mental calculations with increasingly larger numbers.	<p>Mentally- Double any given number with 2 decimal places 38.44 doubled 38 doubled is 76 0.44 doubled is 0.88 Total= 76.88</p>	Decimal point Tenths Hundredths Place value																																				
	Up to 4 digit number multiplied by a 2 digit number using long multiplication	<p>4230 x 27 Estimation 4000</p> <table style="margin-left: auto; margin-right: auto;"> <tr><td></td><td style="text-align: right;">4230</td><td></td></tr> <tr><td></td><td style="text-align: right;">X 27</td><td></td></tr> <tr><td></td><td style="text-align: right;">12000</td><td></td></tr> <tr><td></td><td style="text-align: right;"><u>29610</u> (4230 x 7)</td><td></td></tr> <tr><td></td><td style="text-align: right;"><u>+84600</u></td><td></td></tr> <tr><td></td><td style="text-align: right;"><u>114210</u></td><td></td></tr> </table> <p>Is my estimation close to my answer?</p>		4230			X 27			12000			<u>29610</u> (4230 x 7)			<u>+84600</u>			<u>114210</u>																			
	4230																																					
	X 27																																					
	12000																																					
	<u>29610</u> (4230 x 7)																																					
	<u>+84600</u>																																					
	<u>114210</u>																																					

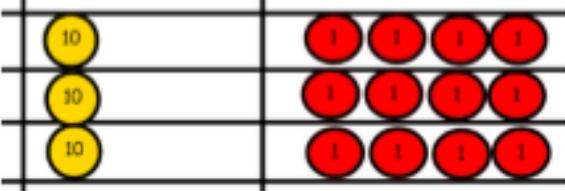
	Multiply both proper and improper fractions	$\frac{2}{5} \times \frac{6}{7} = \frac{2 \times 6}{5 \times 7} = \frac{12}{35}$ $\frac{1}{4} \times \frac{2}{3} = \frac{1 \times 2}{4 \times 3} = \frac{2}{12} = \text{reduces to } \frac{1}{6}$	Multiply, Divide, denominator numerator vinculum
--	---	---	--

Progression in Division

Year Group	Strategy	Examples	Vocabulary
Year 1	Sharing with concrete materials and recording using pictures	<p>Share the buttons between the teddy bears equally.</p>  <p>Can you share the 9 buttons out between the 3 bears fairly? Draw the buttons onto the pictures of three bears?</p> <p>How many buttons does each bear have?</p>	Share, fair, each, same, equal(ly)
	Grouping with concrete materials and record using pictures.	 <p>Can you group these cows so there are four in each field?</p>	Group, lots of, how many groups of
Year 2	Pre-multiplication/division Understand counting backwards in equal steps or jumps 12 - 2 - 2 - 2 - 2 - 2 - 2 - 2	<p>There are twelve wheels, how many bicycles?</p> 	Jump backwards in groups of ..., how many,

		$12 - 2 - 2 - 2 - 2 - 2 - 2$ There are six bicycles.	
	Pictorial (bar model) allow the problem to be represented as an image.	Mr Siddique shares £18 equally between his three sons. How much does each son get?  18 is therefore shared equally as represented above. 	Share equally. Total divided by... Repeated subtraction
Year 2 / 3	Grouping using structured number line to represent jumping back in equal steps to zero with remainders.	I have 22 cakes; I can fit 5 cakes in a box. How many boxes will I need? How many cakes left over?  	Left over, remainder $22 \div 5 = 4$ boxes with 2 cakes left over
	Understand the link between jumping along the number line in equal groups and rotating the number line through 90° and chunking in equal groups.	A cat has eighteen treats. If he is given three treats each day, how many days will the treats last?  $18 - 3 - 3 - 3 - 3 - 3 - 3$  $18 - 3 - 3 - 3 - 3 - 3 - 3 = 6$ The treats will last 6 days as we have taken away 3, six times.	Number line, vertical number line, chunking \div , divide, <i>division</i> , divided by, divided into left, left over, <i>remainder</i>

Year 3	Halve numbers to calculate larger multiple	Children should use dividing by 2 as a strategy to half a number Half of 28 = $28 \div 2 = 14$	Half Groups of share, share
	Sharing using place value counters.	 <p>$42 \div 3 = 14$</p>	
	Divide by chunking on a number line.	<p>Children transfer their knowledge of a vertical number line and begin to take larger chunks away. Jim shared his</p>  $\begin{array}{r} 154 \\ 140 \quad (7 \times 20) \\ \hline 14 \quad (7 \times 2) \\ 14 \\ \hline 0 \end{array}$	Chunking, multiples, Product
Year 3	Using division method children can apply their knowledge to solve word problems and calculate fractional amounts	<p>A computer game is £24 in the sale. This is one quarter off its original price. How much did it cost before the sale?</p>  <p>Children should link their knowledge of $\frac{1}{4}$ meaning shared by 4 to create 4 parts of the whole. 24 divided by 4 = 6 24 - 6 = 18</p>	Part Whole
Year 4	Chunking method dividing by single digit number	<p>Eighty one stickers are divided equally between three friends. How many stickers do they get each? $81 \div 3 =$</p> $\begin{array}{r} 3 \overline{)81} \\ - 30 \quad (10 \times 3) \quad (\text{Chunk from what you know}) \end{array}$	Chunking, multiples, lots of, times tables, use what you know

		$\begin{array}{r} 51 \\ - 30 \quad (10 \times 3) \\ \hline 21 \\ - 21 \quad (7 \times 3) \\ \hline 0 \end{array}$ <p>So $27 \times 3 = 81$ or $81 \div 3 = 27$ Each person gets 27 stickers. Check- $27 \times 3 = 81$</p>	
Year 4	<p>Chunking method with remainders</p>	<p>560 bananas are ordered for 24 monkeys. The monkeys only eat whole bananas. How many bananas do they get each? Are there any left over? Calculation: $560 \div 24 =$</p> $\begin{array}{r} 24 \overline{)560} \\ - 240 \quad (10 \times 24) \quad (\text{Chunk using what you know}) \\ \hline 320 \\ - 240 \quad (10 \times 24) \\ \hline 80 \\ - 48 \quad (2 \times 24) \\ \hline 32 \\ - 24 \quad (1 \times 24) \\ \hline 8 \end{array}$ <p>So $560 \div 24 = 23 \text{ r } 8$ Each monkey gets 23 bananas. There are 8 bananas left over. Checking $552 \div 23 = 24$ and 8 left over</p>	<p>Remainder</p>
Year 4	<p>Bar model to calculate several fractional amounts of a total</p>	<p>Kelly buys four fifths of the shop's oranges. If the shop had 20 oranges, how many does she have?</p>  <p>Whole = 20 parts = 5 amount in each part = ? $20 \div 5 = 4$</p>  <p>4 parts are needed due to it being four fifths so $4 \times 4 = 16$</p>	<p>Whole Multiply Add Share equally divide</p>
Year 5	<p>Begin to use short multiplication identifying the subtracted amounts.</p>	<p>Use place value counters to divide using the bus stop method alongside</p>  <p>$42 \div 3 =$ Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over</p> <p>We exchange this ten for ten ones and then share the ones equally among the groups</p>	

I have 576 balloons for a party. There are 9 families attending. How many balloons will each family be allowed to take?

$$\begin{array}{r} 064 \\ 9 \overline{)576} \\ \underline{57} \\ 36 \\ \underline{36} \\ 0 \end{array}$$

Year 5

Short division requires a up to a 4 digit number to be divided by a 1 digit number

Sarah's gran gave her £2532 and she decided to share it amongst herself and her 5 children. How much money did they each receive

$$\begin{array}{r} \text{Quotient} \\ \text{Divisor } \overline{) \text{ Dividend}} \end{array} \quad \begin{array}{r} 422 \\ 6 \overline{)2532} \end{array}$$

How many 6 in 25? 4, this becomes part of the quotient with 1 remaining. The 1 then moves to the 3 making 13 tens. How many 6 in 13? 2 this becomes the tens part of the quotient with 1 remaining. Giving 12 units. How many 6s in 12 =2.

$$\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{)432} \\ \underline{40} \\ 32 \\ \underline{30} \\ 2 \end{array}$$

Move on to short division with remainders

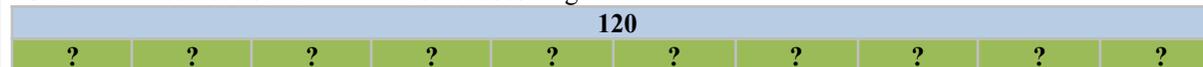
$$\begin{array}{r} 14.6 \\ 35 \overline{)511.0} \\ \underline{35} \\ 16 \\ \underline{15} \\ 11 \\ \underline{10} \\ 10 \\ \underline{10} \\ 0 \end{array}$$

Followed by decimals

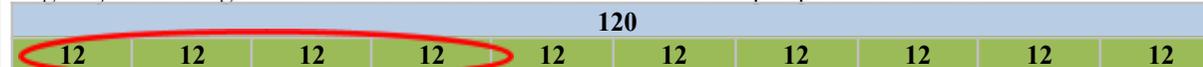
÷, divide, *division*, divided by, divided into, left, left over, *remainder* Quotient

Bar model allows children to represent the whole and identify a percentage of this.

. Sam calculated 40% of 120. What answer does he get?



Begin by calculating 10%. The bar model allows children to see 10 equal parts of 120 = 12

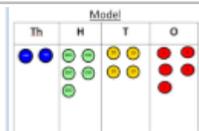


4 lots are needed to create 40% $4 \times 12 = 48$

Year 6

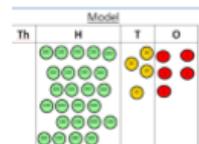
Traditional long division – divide numbers up to 4 digits by a two-digit whole number and interpret remainders as whole number remainders, fractions or by rounding, as appropriate for the context.

Use written division methods in cases where the answer has two decimal places.



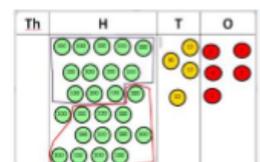
$2544 \div 12$
How many groups of 12 thousands do we have?
None

Exchange 2 thousand for 20 hundreds.



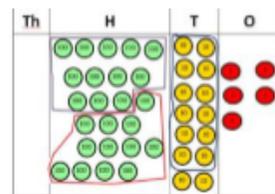
$$12 \overline{) 2544}^0$$

How many groups of 12 are in 25 hundreds? 2 groups. Circle them. We have grouped 24 hundreds so can take them off and we are left with one.



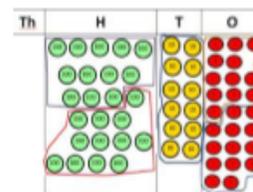
$$12 \overline{) 2544}^{02} \\ \underline{24} \\ 1$$

Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2



$$12 \overline{) 2544}^{021} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2$$

Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2



$$12 \overline{) 2544}^{0212} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0$$

Long division, goes into, goes, remainder, fraction, decimal

There are 552 biscuits altogether in a catering pack.
How many packets of 24 biscuits can be made from the catering pack?

Start dividing 24 into 552.
24 into 5 will not 'go'. 24 into 55 goes twice. Record the 2 on the top line above the second 5. Record the 48 underneath the 55 and subtract, giving 7.
Bring the 2 down and record with the 7 giving 72.
24 into 72 goes three times. Record the 3 on the top line above the 2. Record the 72 underneath the 72 and subtract. Answer zero so no 'remainder'.

$$\begin{array}{r} 23 \\ 24 \overline{) 552} \\ \underline{-48} \\ 72 \\ \underline{-72} \\ 0 \end{array}$$

$552 \div 24 = 23$ There will be 23 packets of 24 biscuits

There are 558 biscuits. How many packets of 24?

		$ \begin{array}{r} 23.25 \\ 24 \overline{) 558.00} \\ \underline{48} \\ 78 \\ \underline{72} \\ 60 \\ \underline{48} \\ 180 \\ \underline{180} \\ 0 \end{array} $ <p>There will be 23 packets with 0.25 of a packet left over or 6 biscuits left over or $6/24 = 1/4$ left over.</p>									
Divide numbers with up to two decimal places by one-digit and then two-digit whole numbers		<p>A piece of wood measuring 6.75m is sawn into 5 equal sections. How long is each section?</p> $ \begin{array}{r} 1.35 \\ 5 \overline{) 6.75} \end{array} $ <p>Each section will measure 1.35m.</p>	Long division, remainder								
The bar model allows us to calculate different fraction parts and more complex word problems which require more than one step.		<p>Three quarters of a number is 54. What is the number?</p> <table border="1" data-bbox="660 502 1870 571"> <tr> <td colspan="3" style="background-color: #d9e1f2; text-align: center;">54</td> <td style="background-color: #d9e1f2; text-align: center;">1/4</td> </tr> <tr> <td style="background-color: #c6e0b4; text-align: center;">?</td> </tr> </table> <p>By demonstrating in the bar model we can identify the whole by first finding $1/4$ and then multiplying this by 4. $54 \div 3 = 18$ $18 \times 4 = 72$</p>	54			1/4	?	?	?	?	
54			1/4								
?	?	?	?								